Name solutions

ECE 311 Exam 3 Fall 2008

December 4, 2008

Closed Text and Notes

1) Be sure you have 13 pages.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) Write neatly, if your writing is illegible then print.

4) The last 5 pages contain equations that may be of use to you.

5) This exam is worth 100 points.

(6 pts) 1. Identify the statement (only one) that is not true of ferromagnetic materials

A) They have a large χ_m

B) They have a fixed value of μ_r

C) They retain a considerable amount of their magnetization when removed from an external magnetic field.

D) They loose their nonlinearity property above the curie temperature

(6 pts) 2. For static fields, $\int_A^B \mathbf{E} \bullet \mathbf{dl}$ is independent of the path

A) only if **E** is spatially uniform.

B) only if **E** has spherical symmetry.

C) only if **E** is due to a uniform line charge.

D) always.

(6 pts) 3. A current of 1 A is flowing in a circular wire of radius 0.1 m. If the fingers of one's right hand are curled in the direction of the current in the loop, then one's thumb points in the direction $\hat{a}_m$. If the loop is free to rotate, what direction would $\vec{a}_m$ point if the loop is placed in a uniform magnetic field of $\vec{B} = 100\ \hat{a}_y$ T?

A) $\vec{a}_m = \hat{a}_x$

B) $\vec{a}_m = -\hat{a}_x$

C) $\vec{a}_m = \hat{a}_y$

D) $\vec{a}_m = \hat{a}_z$

(6 pts) 4. A 10^{-9} C charge is moving with velocity $u = 2x10^5 \hat{a}_x \frac{m}{s}$ in the presence of uniform electric and magnetic fields. The values of the electric and magnetic fields are, $\vec{E} = 10^6\ \hat{a}_y \frac{V}{m}$ and $\vec{B} = 10\ \hat{a}_z$ T. The vector force exerted on the moving charge due to the fields is

A) $-2x10^{-3}\ \hat{a}_y$ N

B) $-1x10^{-3}\ \hat{a}_y$ N

C) $+2x10^{-3}\ \hat{a}_y$ N

D) $-3x10^{-3}\ \hat{a}_y$ N

$$\vec{F} = q(\vec{E} + \vec{u} \times \vec{B})$$
$$= 10^{-9} C \left[10^6 \hat{a}_y \frac{V}{m} + (2x10^5 \tfrac{m}{s} \hat{a}_x) \times 10 \hat{a}_z \frac{Wb}{m^2}\right]$$
$$= 10^{-9} C \left[10^6 \hat{a}_y \frac{V}{m} + 2x10^6 (-\hat{a}_y) \frac{V}{m}\right]$$
$$= 10^{-9} C \left[-10^6 \hat{a}_y \frac{V}{m}\right)$$
$$= -10^{-3} N \hat{a}_y$$

(6 pts) 5.The region for y < 0 has relative permeability of 1 and the region for y > 0 a relative permeability of 3. If $H = 6\hat{a}_x \frac{A}{m}$ for y < 0 and there is a sheet current density of $K = 5\hat{a}_z \frac{A}{m}$ flowing on the in the y = 0 plane, the magnetic field intensity for y > 0 is

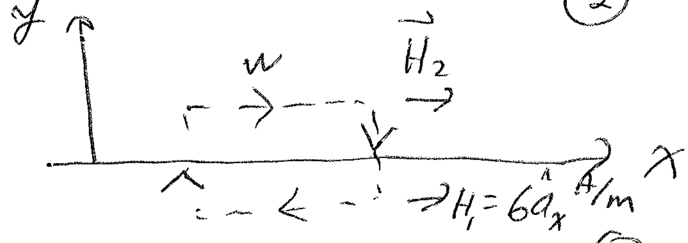
A) $H = 18\hat{a}_x \frac{A}{m}$

B) $H = 6\hat{a}_x \frac{A}{m}$

C) $H = 11\hat{a}_x \frac{A}{m}$

(D) $H = 1\hat{a}_x \frac{A}{m}$

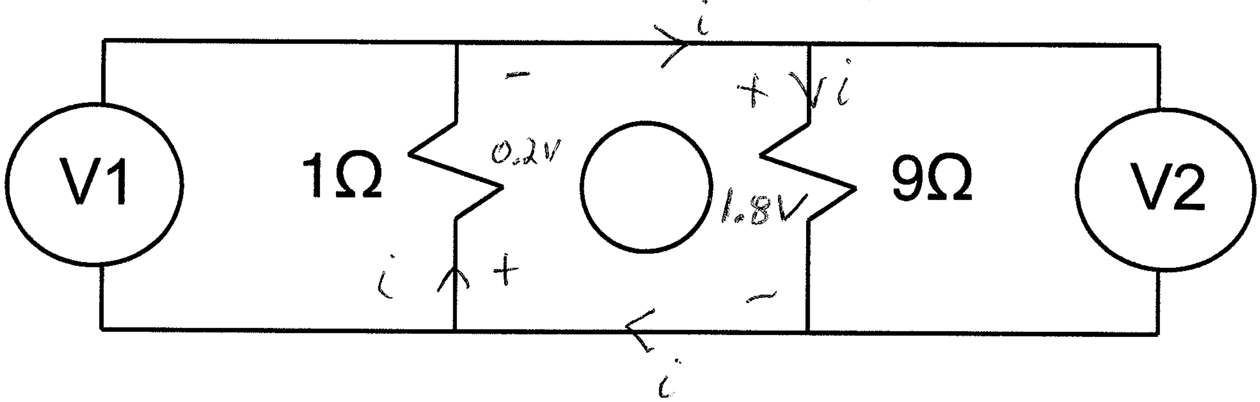
E) $H = 23\hat{a}_x \frac{A}{m}$



$H_2 w - 6\frac{A}{m} w = -5\frac{A}{m} w$

$H_2 = 1 \frac{A}{m} \hat{a}_x$

(12 pts) 6. Shown is a circuit consisting of two resistors. Through the circuit loop consisting of the two resistors is a solenoid indicated by a circle. A current through the solenoid produces a flux of $\psi = 2tWb$ out of the page. What are the readings on the two voltmeters shown, V1 and V2? Assume the positive terminals of both V1 and V2 are at the top of the circuit.

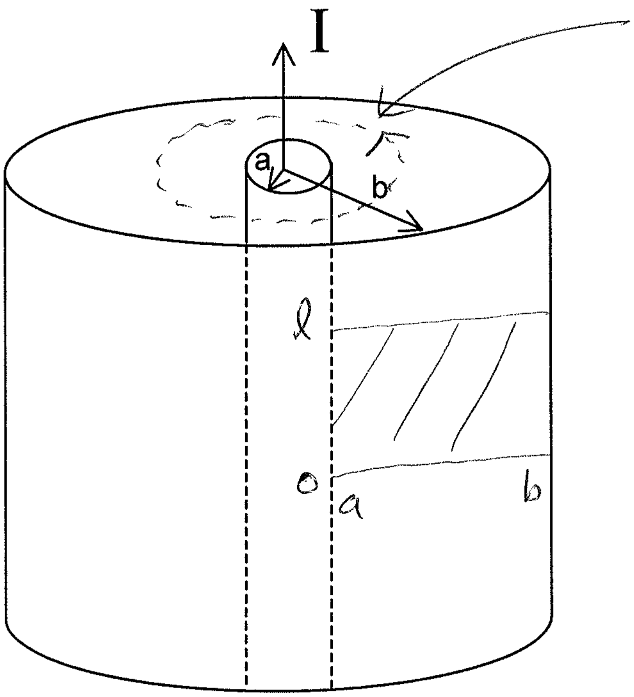


$\frac{d\psi}{dt} = \frac{d}{dt}(2t\ Wb) = 2V$

$i = \frac{2V}{10\Omega} = 0.2A$ in the clockwise direction as indicated on the figure

so, $V_2 = 1.8V$ $V1 = -0.2V$

(15 pts) 7. Derive the inductance per unit length for the coaxial cable shown. The inner conductor has radius a and the outer conductor radius b. The outer conductor is grounded.



$$\oint H \cdot dl = I$$

$$\int_0^{2\pi} H \hat{a}_\phi \cdot \rho \, d\phi \, \hat{a}_\phi = I$$

$$\int_0^{2\pi} H \rho \, d\phi = I$$

$$H \rho \, 2\pi = I$$

$$\vec{H} = \frac{I}{2\pi\rho} \hat{a}_\phi$$

$$\psi = \int \vec{B} \cdot \vec{ds} = \mu_0 \int \vec{H} \cdot \vec{ds} = \mu_0 \int_0^l \int_a^b \frac{I}{2\pi\rho} \hat{a}_\phi \cdot d\rho \, dz \, \hat{a}_\phi$$

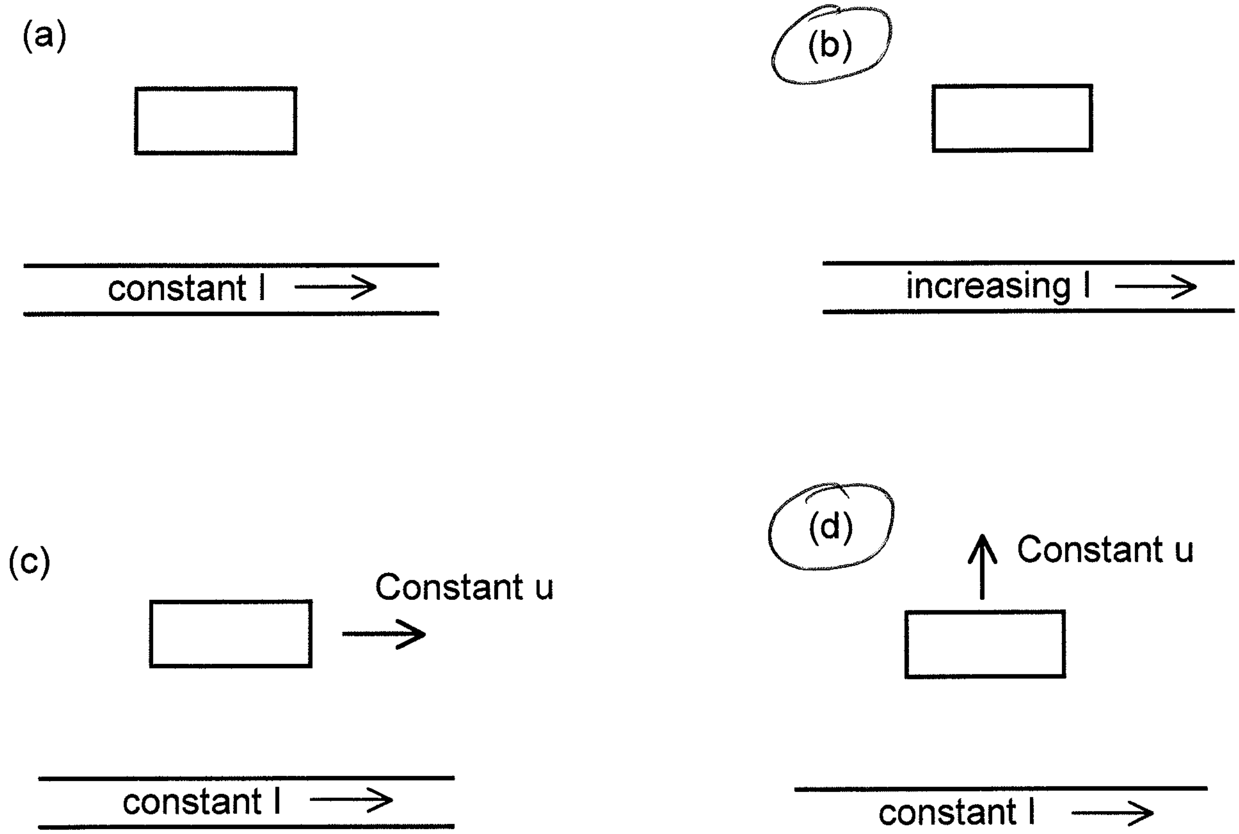
$$\psi = \frac{\mu_0 I}{2\pi} \int_0^l \int_a^b \frac{1}{\rho} d\rho \, d\phi = \frac{\mu_0 l I}{2\pi} \int_a^b \frac{d\rho}{\rho}$$

$$\psi = \frac{\mu_0 l I}{2\pi} \ln \rho \Big|_a^b = \frac{\mu_0 l I}{2\pi} \ln \frac{b}{a}$$

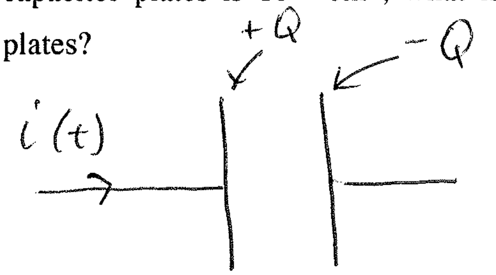
$$L = \frac{\psi}{I} = \frac{\mu_0 l}{2\pi} \ln \frac{b}{a}$$

inductance per unit length $= \dfrac{L}{l} = \dfrac{\mu_0}{2\pi} \ln \dfrac{b}{a}$

(6 pts) 8. Which of the following four rectangular conducting loops will have currents flowing in them? Note it might be zero, one, two, three, or all four of the configurations.



(10 pts) 9. A parallel plate capacitor is being charged by a current of value i(t) = 10t A. If the area of the capacitor plates is 10^{-4} cm^2, what is the displacement current density between the capacitor plates?

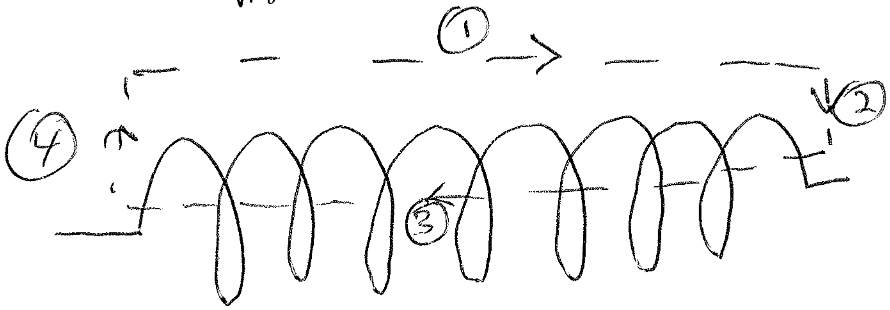


$$DA = Q$$

$$J_d = \frac{\partial D}{\partial t} = \frac{1}{A}\frac{dQ}{dt} = \frac{1}{A} i(t)$$

$$J_d = \frac{10t\ A}{10^{-4} cm^2} = 10^5 t \frac{A}{cm^2}$$

(10 pts) 10. A tightly wound coil of 1000 turns is 0.1 m long, has free space inside the coil, and has a current of $\frac{1}{\sqrt{\mu_o}}$ A flowing through it. What is the energy density inside the coil?



$$\oint \vec{H} \cdot d\vec{l} = I_{enclosed} = 1000 \frac{1}{\sqrt{\mu_o}}$$

$\int \vec{H} \cdot d\vec{l}$ along ①, ②, + ④ is small compared to $\int \vec{H} \cdot d\vec{l}$ on ③ so

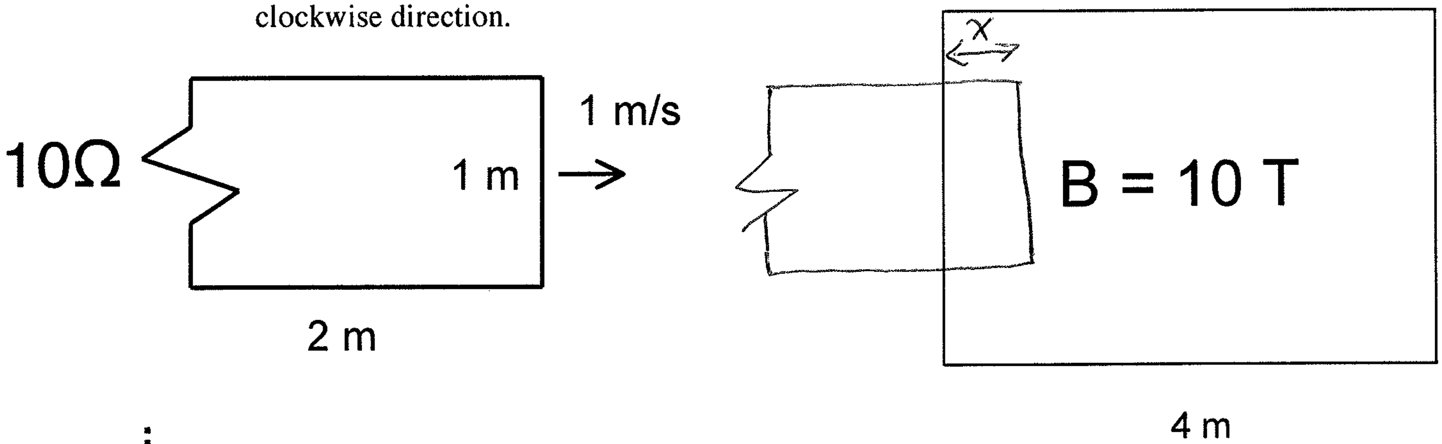
$$\int_{③} \vec{H} \cdot d\vec{l} = H(0.1m) = 1000 \frac{1}{\sqrt{\mu_o}}$$

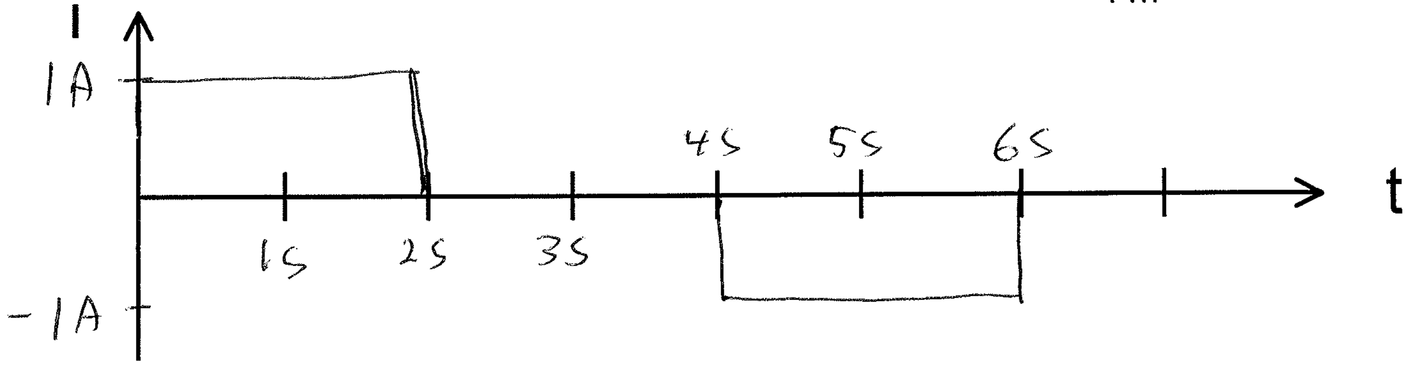
$$H = \frac{10^4}{\sqrt{\mu_o}}$$

$$\text{energy density} = \frac{1}{2}\mu_o H^2 = \frac{1}{2}\mu_o \left(\frac{10^4}{\sqrt{\mu_o}}\right)^2$$

$$= 5 \times 10^7 \frac{J}{m^3}$$

(15 pts) 11. A 2m x 1m rectangular loop contains a 10 Ω resistor and is moving at 1 m/s towards a region of uniform magnetic field 10 T out of the page and of width 4 m. Assume the rectangular loop begins to enter the 4 m wide region of uniform magnetic field at t=0. Plot the current versus time in the 10 Ω resistor from t = 0 to t = 7 s. Assume the current is positive if flowing in a clockwise direction.





$x = ut = (1\frac{m}{s})t$ $\qquad A = x(2m) = 1\frac{m^2}{s}t$

$\psi = AB = (1\frac{m^2}{s}t)\,10\frac{Wb}{m^2} = 10t\ V$

$\frac{d\psi}{dt} = 10V \qquad i = \frac{10V}{10\Omega} = 1A$ from $t = 0$ to $2sec$

once the loop is completely in the magnetic field $\frac{d\psi}{dt} = 0$ so $i = 0$

as the loop exists the magnetic field, from $t = 4s$ to $6s$, $\frac{d\psi}{dt} = -10V$

so $i = -1A$